64. What's In a Name? The word *sine* comes from the Latin word *sinus*, which means "bay" or "cove." It entered the language through a mistake (variously attributed to Gerardo of Cremona or Robert of Chester) in translating the Arabic word "jiba" (chord) as if it were "jaib" (bay). This was due to the fact that the Arabs abbreviated their technical terms, much as we do today. Imagine someone unfamiliar with the technical term "cosecant" trying to reconstruct the English word that is abbreviated by "csc." It might well enter their language as their word for "cascade."

The names for the other trigonometric functions can all be explained.

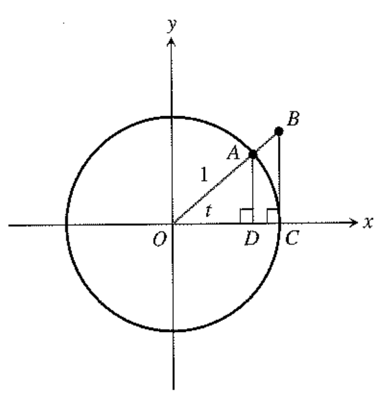
(a) *Cosine* means "*sine* of the *co*mplement." Explain why this is a logical name for cosine.

(b) In the figure below, BC is perpendicular to OC, which is a radius of the unit circle. By a familiar geometry theorem, BC is tangent to the circle. OB is part of a secant that intersects the unit circle at A. It lies along the terminal side of an angle of t radians in standard position. Write the coordinates of A as functions of t.

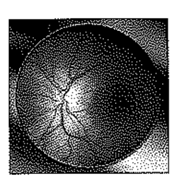
(c) Use similar triangles to find length BC as a trig function of t.

(d) Use similar triangles to find length OB as a trig function of t.

(e) Use the results from parts (a), (c), and (d) to explain where the names "tangent, cotangent, secant," and "cosecant" came from.



65. Capillary Action A film of liquid in a thin (capillary) tube has surface tension γ (gamma) given by



$$\gamma = \frac{1}{2}h\rho g r \sec\phi,$$

where h is the height of the liquid in the tube, ρ (rho) is the densi-ty of the liquid, $g = 9.8\ \text{m/sec}^2$ is the acceleration due to gravity, r is the radius of the tube, and ϕ (phi) is the angle of contact between the tube and the liquid's surface. Whole blood has a surface tension of 0.058 N/m (newton per meter) and a density of 1050 kg/m^3. Suppose that blood rises to a height of 1.5 m in a capillary blood vessel of radius 4.7×10^{-6} m. What is the contact angle between the capillary vessel and the blood surface? ($1\ \text{N} = 1\ (\text{kg}\cdot\text{m})/\text{sec}^2$)

66. Advanced Curve Fitting A researcher has reason to believe that the data in the table below can best be described by an algebraic model involving the secant function:

$$y = a \sec (bx).$$

Unfortunately, her calculator will only do sine regression. She realizes that the following two facts will help her:

$$\frac{1}{y} = \frac{1}{a\sec(bx)} = \frac{1}{a}\cos(bx)$$

and

$$\cos(bx) = \sin\left(bx + \frac{\pi}{2}\right).$$

(a) Use these two facts to show that

$$\frac{1}{y} = \frac{1}{a}\sin\left(bx + \frac{\pi}{2}\right).$$

(b) Store the x values in the table in L1 in your calculator and the y values in L2. Store the *reciprocals* of the y values in L3. Then do a sine regression for L3 $(1/y)$ as a function of L1 (x). Write the regression equation.

(c) Use the regression equation in (b) to determine the values of a and b.

(d) Write the secant model: $y = a \sec (bx)$. Does the curve fit the (L1, L2) scatter plot?

x	1	2	3	4
y	5.0703	5.2912	5.6975	6.3622

x	5	6	7	8
y	7.4359	9.2541	12.716	21.255

4.6 Graphs of Composite Trigonometric Functions

What you'll learn about

- Combining Trigonometric and Algebraic Functions
- Sums and Differences of Sinusoids
- Damped Oscillation

. . . and why

Function composition extends our ability to model periodic phenomena like heartbeats and sound waves.

Combining Trigonometric and Algebraic Functions

A theme of this text has been "families of functions." We have studied polynomial functions, exponential functions, logarithmic functions, and rational functions (to name a few), and in this chapter we have studied trigonometric functions. Now we consider adding, multiplying, or composing trigonometric functions with functions from these other families.

The notable property that distinguishes the trigonometric function from others we have studied is periodicity. Example 1 shows that when a trigonometric function is combined with a polynomial, the resulting function may or may not be periodic.

EXAMPLE 1 Combining the Sine Function With x^2

Graph each of the following functions for $-2\pi \le x \le 2\pi$, adjusting the vertical window as needed. Which of the functions appear to be periodic?

(a) $y = \sin x + x^2$

(b) $y = x^2 \sin x$

(c) $y = (\sin x)^2$

(d) $y = \sin(x^2)$

SOLUTION We show the graphs and their windows in Figure 4.56 on the next page. Only the graph of $y = (\sin x)^2$ exhibits periodic behavior in the interval $-2\pi \le x \le 2\pi$. (You can widen the window to see further graphical evidence that this is indeed the only periodic function among the four.)

Now try Exercise 5.

EXAMPLE 2 Verifying Periodicity Algebraically

Verify algebraically that $f(x) = (\sin x)^2$ is periodic and determine its period graphically.

SOLUTION We use the fact that the period of the basic sine function is 2π, that is, $\sin(x + 2\pi) = \sin(x)$ for all x. It follows that

$$\begin{aligned} f(x + 2\pi) &= (\sin(x + 2\pi))^2 \\ &= (\sin(x))^2 \quad \text{By periodicity of sine} \\ &= f(x) \end{aligned}$$

So $f(x)$ is also periodic, with some period that divides 2π. The graph in Figure 4.56c on the next page shows that the period is actually π.

Now try Exercise 9.

EXPONENT NOTATION

Example 3 introduces a shorthand notation for powers of trigonometric functions: $(\sin\theta)^n$ can be written as $\sin^n\theta$. (Caution: This shorthand notation will probably not be recognized by your calculator.)